Markus Osterhoff:

Coherence in Stationary Stochastic Optical Fields

Coherence
in stationary stochastic optical fields

Markus Osterhoff [a,b], Christian Morawe [b], Tim Salditt [a]

[a] Institut für Röntgenphysik, University of Göttingen;
[b] Multilayer Group, ESRF, Grenoble

SciSoft Coffee Meeting – March, 18th 2010

Outline

Coherent Imaging

Coherent Diffractive Imaging (CDI)

- biological samples:
  weak scattering is stronger than absorption
- reconstruction with resolutions of some ten nanometers possible

- index of refraction for hard x-rays:
  $n = 1 - \delta + i\beta$

- $\delta \lesssim 10^{-4}$ – phase shift
- $\beta \lesssim 10^{-6}$ – absorption

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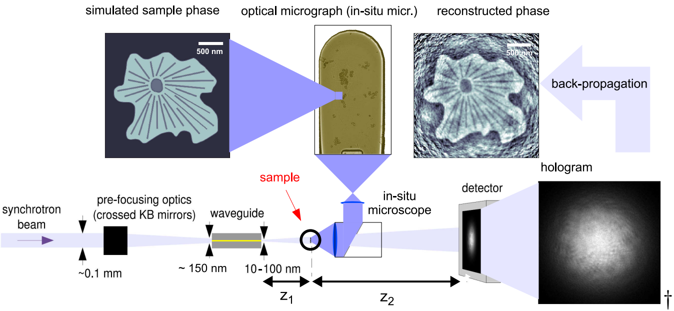

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† K. Giewekemeyer *et al*, NJP (2010, accepted)

Coherent Imaging

Coherent Diffractive Imaging (CDI)

- biological samples:
  weak scattering is stronger than absorption
- reconstruction with resolutions of some ten nanometers possible

- highly coherent illumination needed
- beam shaping optics needed
- knowledge of the illumination function improves reconstruction results

PETRA III

- Coherent Imaging at the nanoscale needs high flux of coherent photons

$\longrightarrow$ need of 3$^{\text{rd}}$ generation synchrotron

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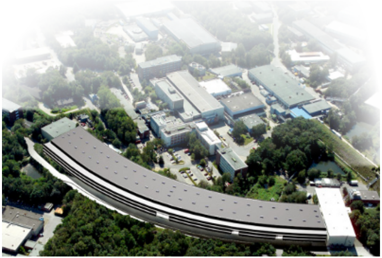
PETRA III @ Hasylab (DESY, Germany)

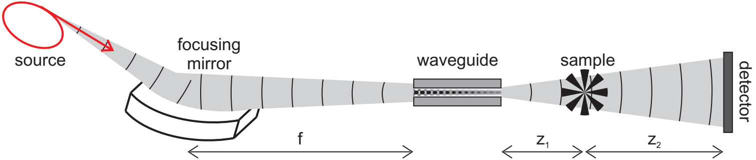
- in october 2009, horizontal emittance of 1 nm rad was reached†
- first beamlines can be used

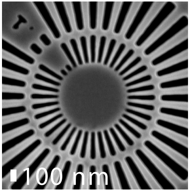


†http://petra3.desy.de

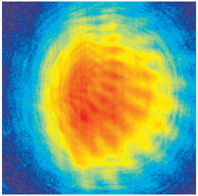
P10 – Coherence Beamline

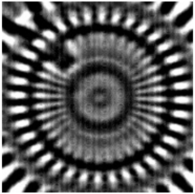
Dedicated setup for waveguide-based coherent imaging operated by IRP @ P10, PETRA III†





specimen


detector signal


reconstruction

mirrors are being aligned *right now*!
– this data: from ESRF, ID22NI

†S. Kalbfleisch, M. Osterhoff *et al*, SRI proceedings (2009, accepted)

The mirrors

- Two total reflection mirrors in a KB scheme
- Mirror overview:

	JTEC	WinlightX
focusing	horizontal	vertical
distance from source	88.5 m	88.4 m
distance to focus	200 mm	300 mm
angle of incidence	4 mrad	3.8 mrad
active length	$\approx$ 94 mm	
material	silica	
coating	palladium	
figure error (P-V) (after coating)	4.8 nm	

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Figure error: perfect mirror

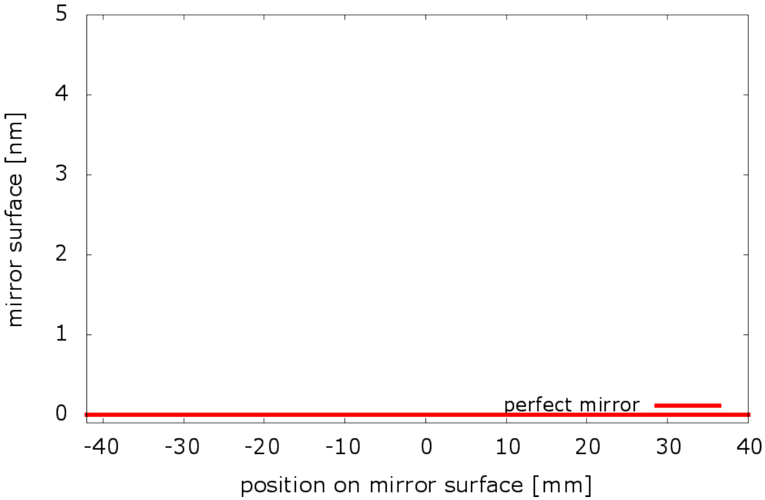


Figure error: polished mirror

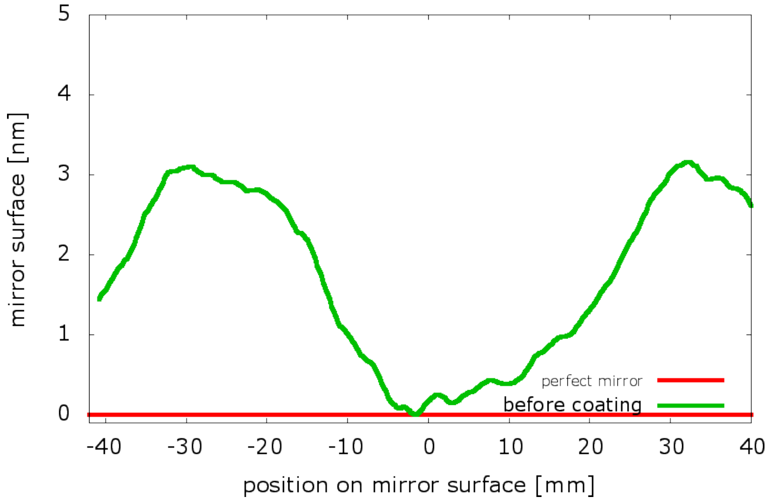
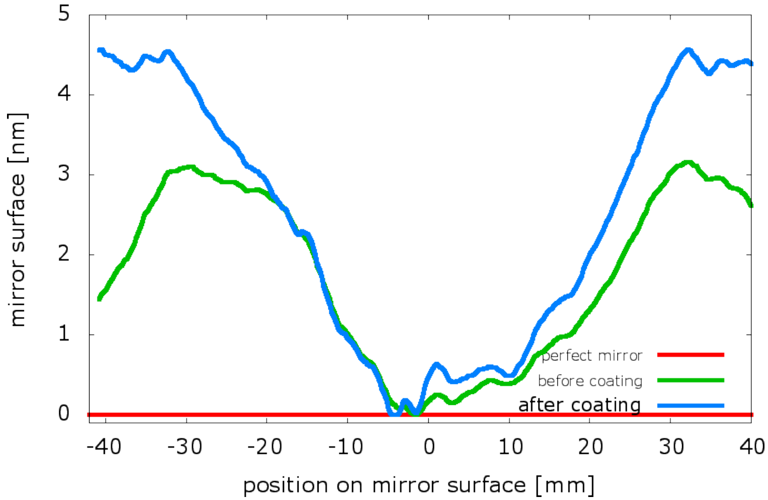


Figure error: coated mirror



- roughness: $\sigma \approx 0.1$ nm $\rightarrow$ not significant

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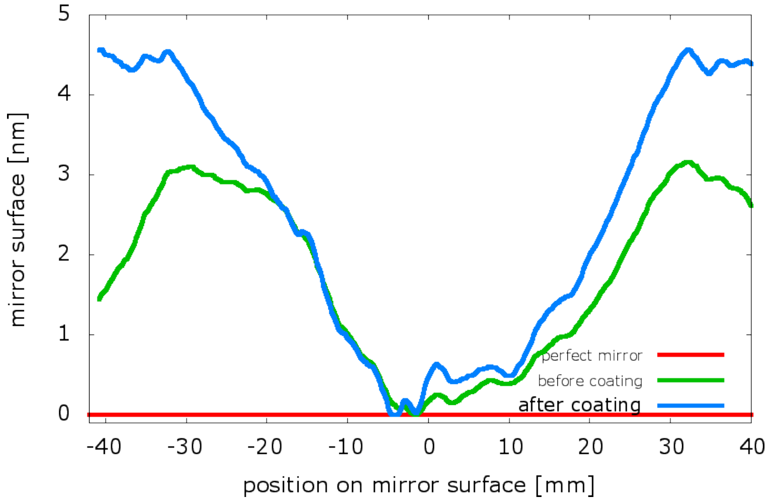

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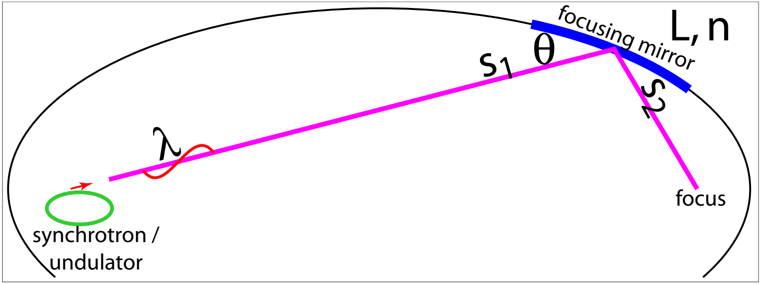
Figure error

- Figure error before/after coating was measured by Frank Siewert @ HZB–Bessy II, Berlin
- "The Nanometer Optical Component Measuring Machine"

- measured figure error of 4.8 nm (p-v) is *very low*;
- NOM's reproducibility is $\pm$ 1.2 nm (p-v)
- error before coating was $\approx$ 3 nm (p-v)

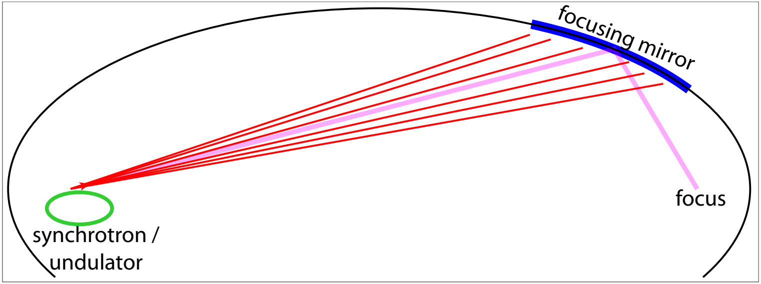
- possible problems: alignment, world-travelling mirror

Simulation



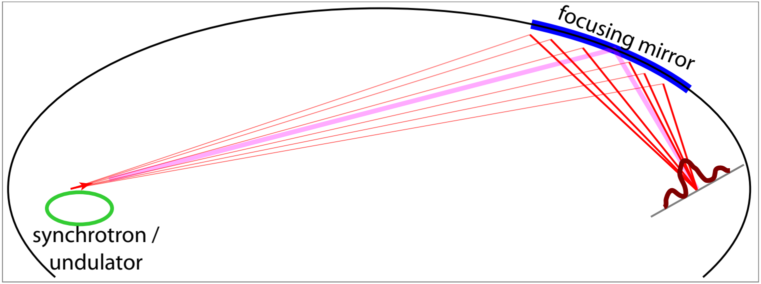
- Mirror performance was calculated by waveoptical methods
- Geometry is defined by s_1, s_2, Θ
- wavelength λ, mirror's length L and $n = 1 - \delta + i\beta$

Simulation



- Phase–ray–tracing from a point-source: $E(x_1) = E_0 \frac{e^{ikr}}{\sqrt{r}}$
- Fresnel's coefficient: $r_\sigma = \frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}$
- complex θ', n include phase shifts

Simulation

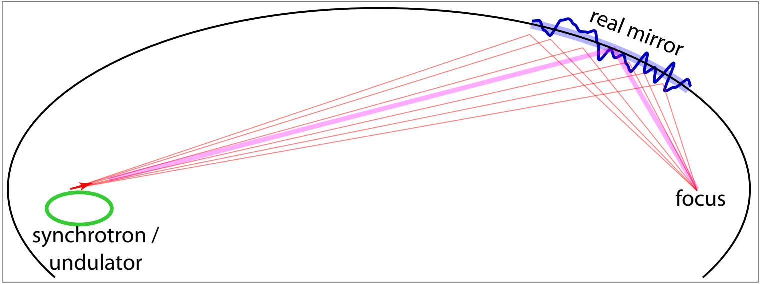


- Kirchhoff's integral of diffraction:

$$E(x_2) = \frac{-i}{\sqrt{\lambda}} \int \mathrm{d}x_1 \; E(x_1) \frac{e^{ikr}}{\sqrt{r}}$$

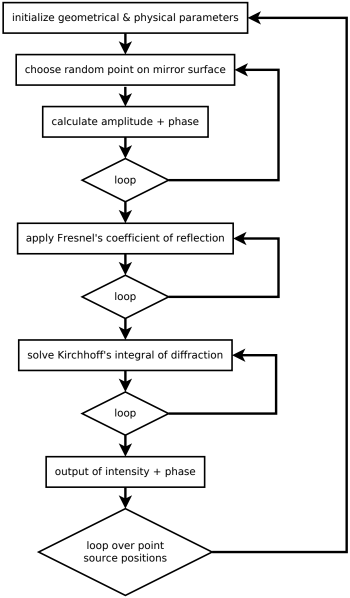
- 1d-detector
- 2d-detector (in propagation direction)

Simulation



- include mirror's real surface profile
  as determined by NOM / metrology

Simulation


initialize geometrical & physical parameters
choose random point on mirror surface
calculate amplitude + phase
loop
apply Fresnel's coefficient of reflection
loop
solve Kirchhoff's integral of diffraction
loop
output of intensity + phase
loop over point source positions

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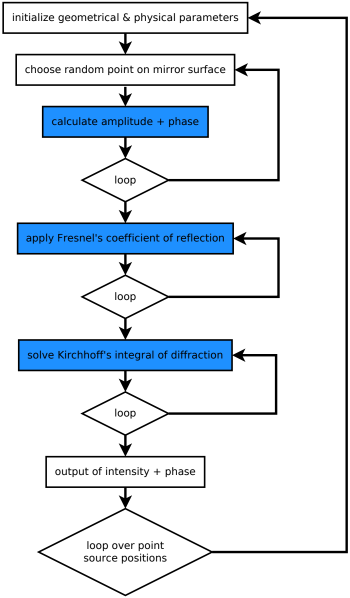
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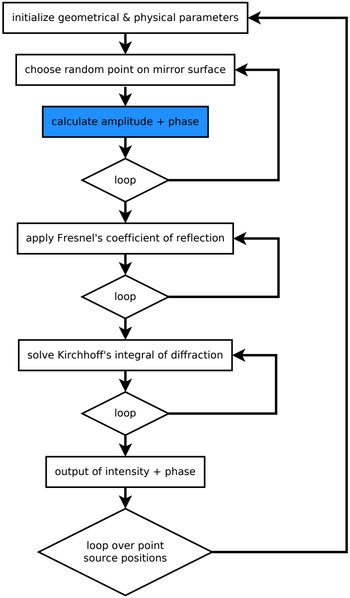
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Focus Fields – behind the scenes

- We need the *phase* on the mirror
- $\lambda \approx 10^{-10}\,\mathrm{m} \qquad \mathrm{s}_1 \approx 10^{2}\,\mathrm{m}$
- 12 orders of magnitude!

- hardware precision (double): 64 bits, 48 are significant
- corresponding to 15 decimal places

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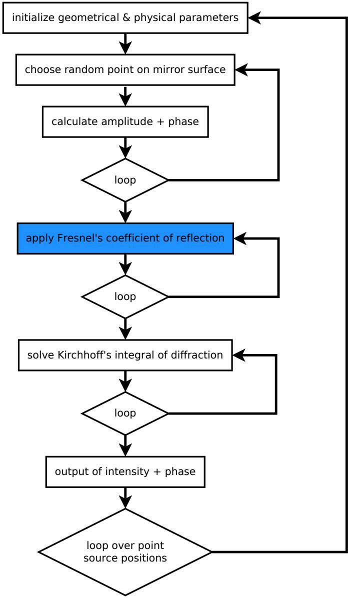
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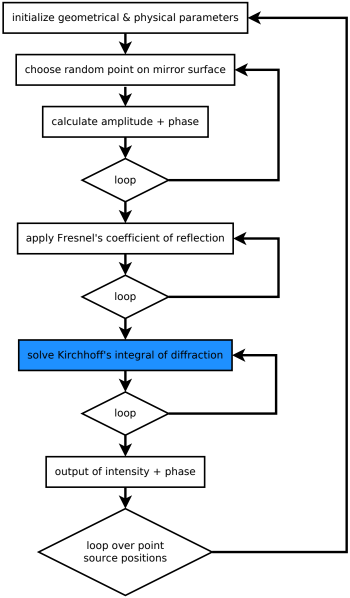
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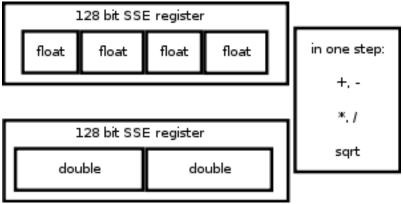

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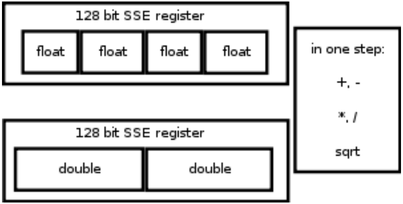

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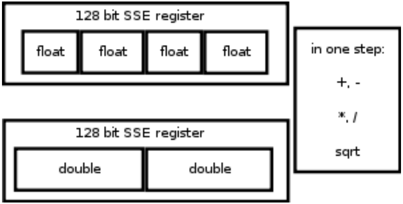

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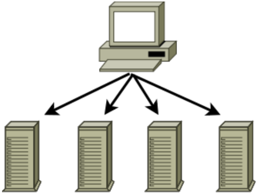

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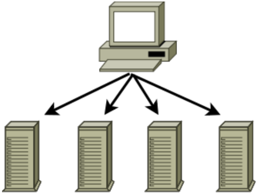

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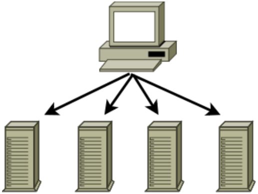

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Simulated fields

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- point source
- PETRA III source

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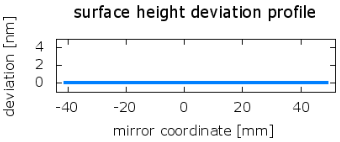
Focus fields – point source + ideal mirror

perfect mirror
peak-to-valley: 0.0 nm



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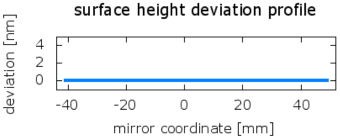
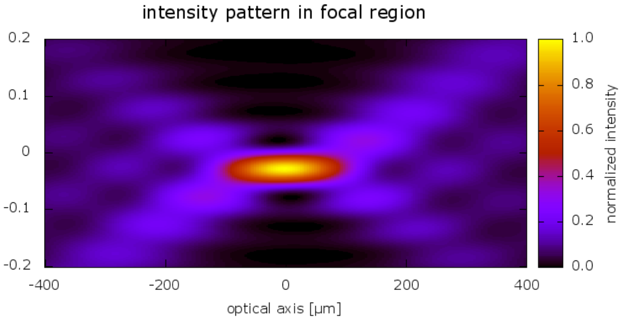
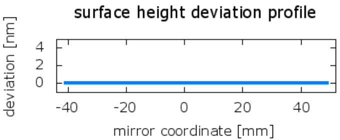
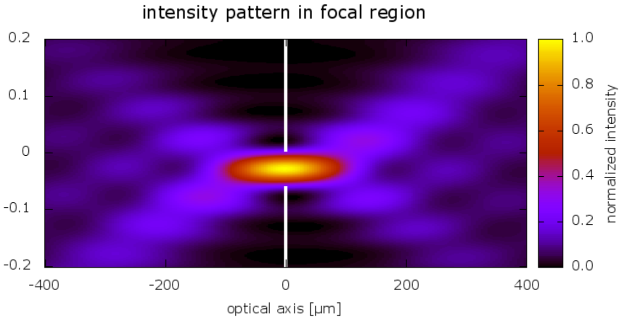




Focus fields – point source + ideal mirror

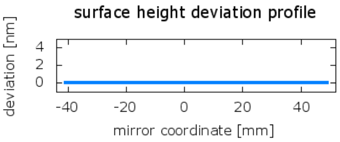
perfect mirror
peak-to-valley: 0.0 nm
focus-width:

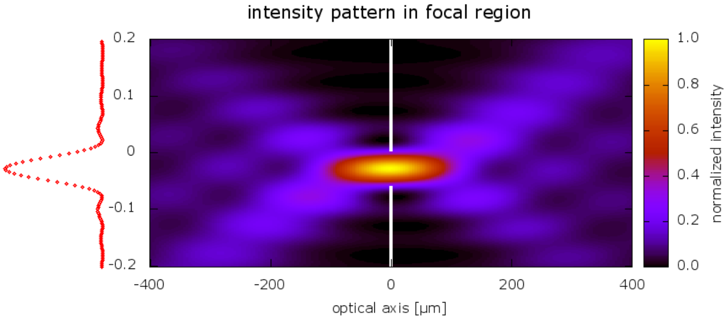




Focus fields – point source + ideal mirror

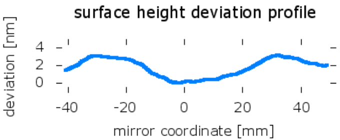
perfect mirror
peak-to-valley: 0.0 nm
focus-width: 54 nm

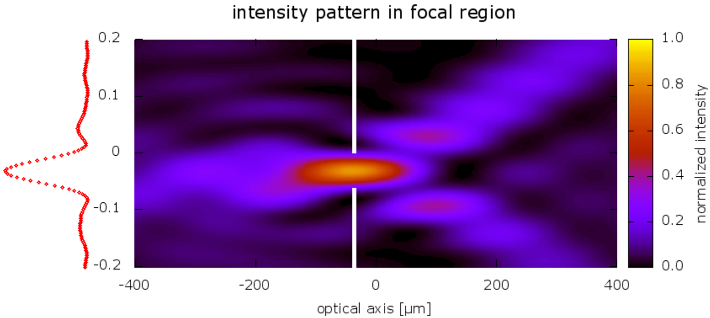




Focus fields – point source + polished mirror

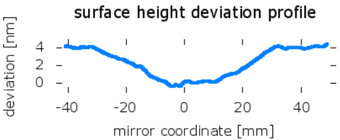
polished mirror
peak-to-valley: 3.1 nm
focus-width: 54 nm

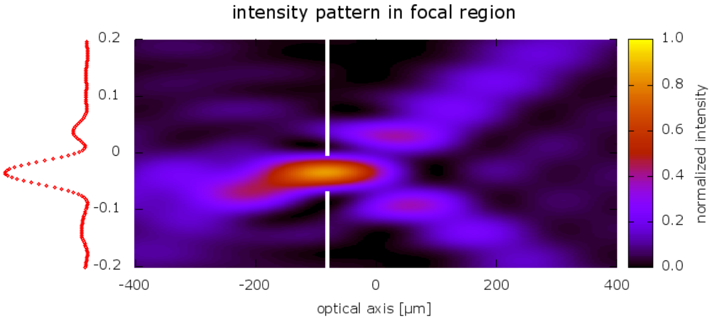




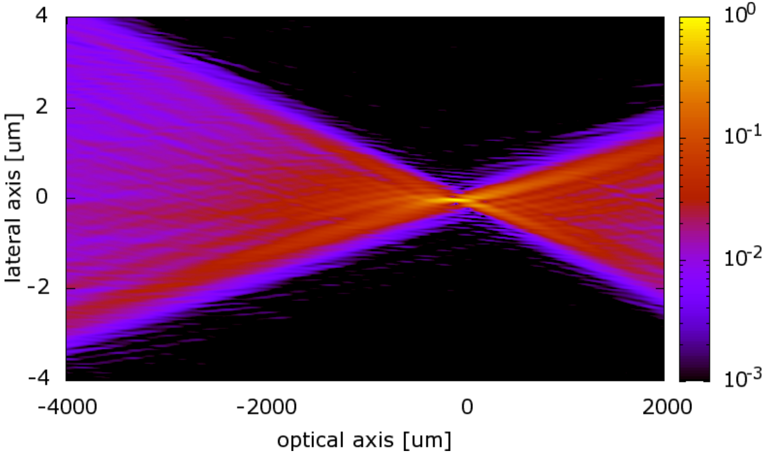
Focus fields – point source + coated mirror

coated mirror
peak-to-valley: 4.8 nm
focus-width: 54 nm





Focus fields – long



Focus fields – extended source + coated mirror

- Source size: 36 μm (1σ)

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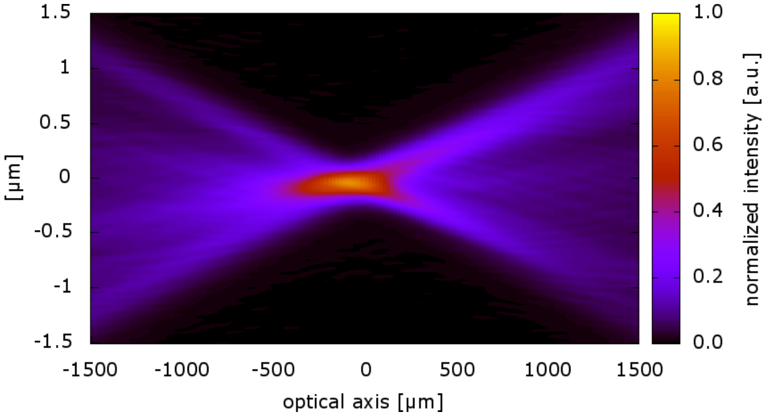

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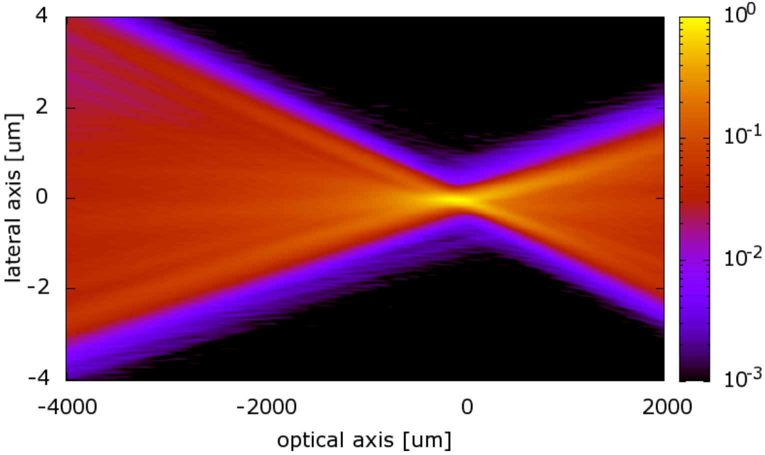
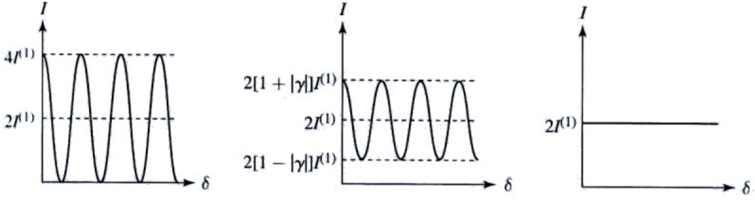


Coherence in the focal region

- complex degree of coherence:

$$\gamma(\vec{r}_1, \vec{r}_2, \tau) = \frac{\langle U(\vec{r}_1, t)U^*(\vec{r}_2, t+\tau)\rangle}{\sqrt{\langle U(\vec{r}_1, t)U^*(\vec{r}_1, t)\rangle\langle U(\vec{r}_2, t)U^*(\vec{r}_2, t)\rangle}}$$



(a) coherent superposition ($|\gamma| = 1$)
(b) partially coherent superposition ($0 < |\gamma| < 1$)
(c) incoherent superposition ($|\gamma| = 0$)

(E. Wolf: Theory of Coherence and Polarization of Light, Fig. 3.2)

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$$\gamma = \frac{\langle U_1 U_2^* \rangle}{\sqrt{I_1 I_2}}$$

- Nature: time-average

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- Nature: time-average
- Simulation: ensemble-average

$$U(x) = \sum_n w_n c_n^{\text{rand}} u_n(x)$$

Coherence in the focal region

- complex degree of coherence:

$$\gamma = \frac{\langle U_1 U_2^* \rangle}{\sqrt{I_1 I_2}}$$

- Nature: time-average
- Simulation: ensemble-average

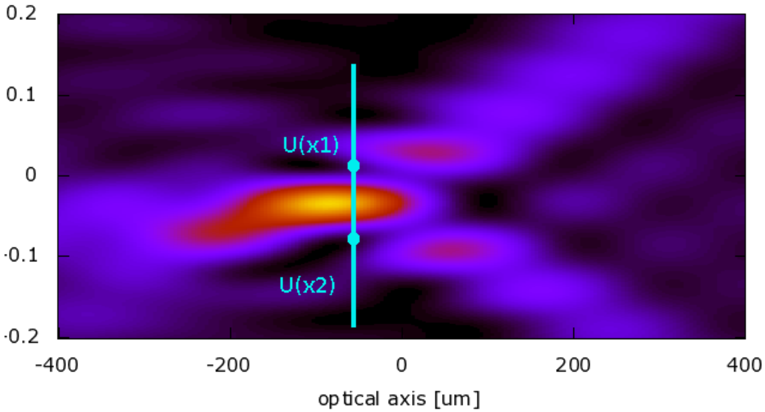
$$U(x) = \sum_n w_n c_n^{\text{rand}} u_n(x)$$

Coherence in the focal region

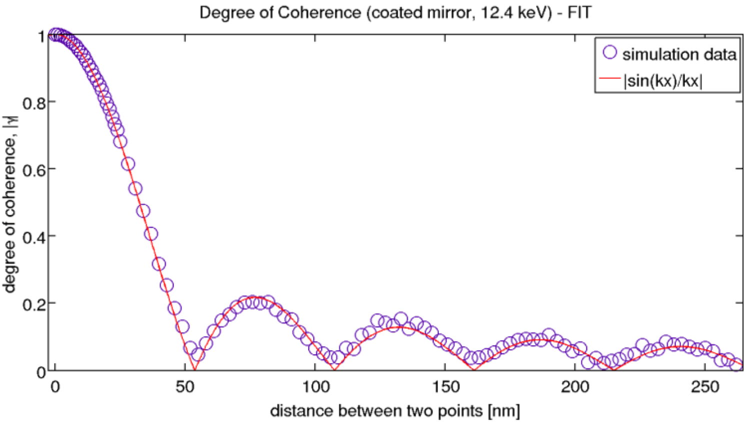
$$U(x) = \sum_n w_n c_n^{\text{rand}} u_n(x)$$

- w_n: weighting coefficients (Gaussian envelope for point-sources)
- c_n^{rand}: random complex coefficients for superposition
- $u_n(x)$: pre-calculated field distributions for nth source

Coherence in the focal region – sketch



Coherence in the focal region – coated mirror



Coherence in the focal region – coated mirror

coherence

M. Osterhoff

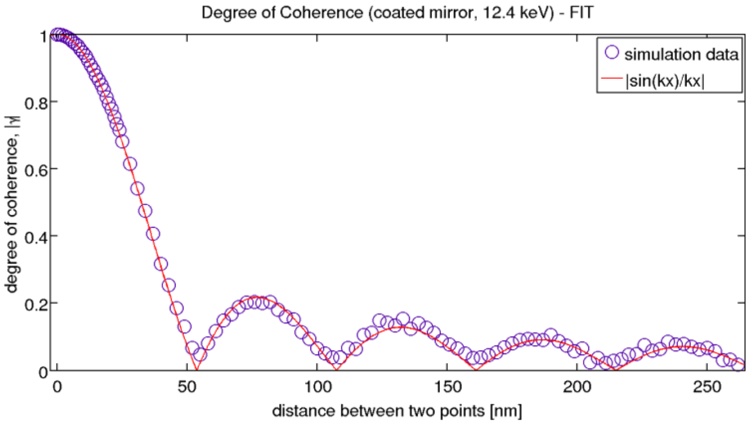
Introduction

Focus

Coherence

3D-Focus

Summary



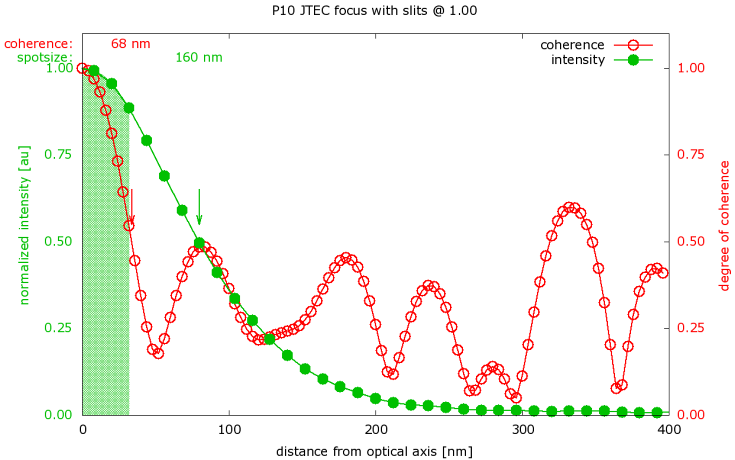
- Coherence length is ≈ 54 nm
  – corresponding to mirror's aperture

Coherent Flux

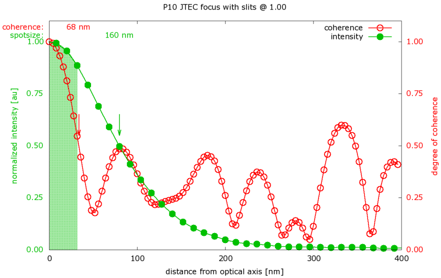
- relation between coherence and intensity?

Coherent Flux

- relation between coherence and intensity?



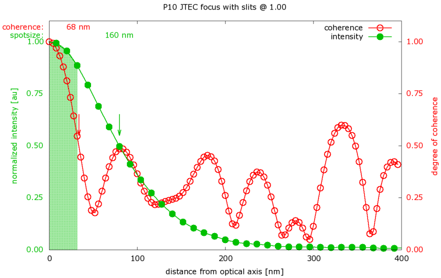
Coherent Flux



slits in front of the mirror:

- less intensity

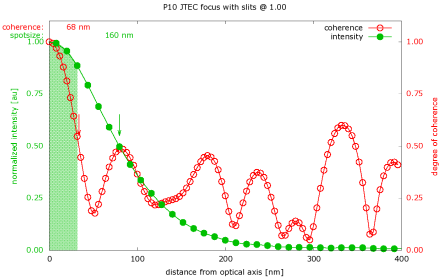
Coherent Flux



slits in front of the mirror:

- less intensity
- smaller NA $\longrightarrow$ larger spot

Coherent Flux



slits in front of the mirror:

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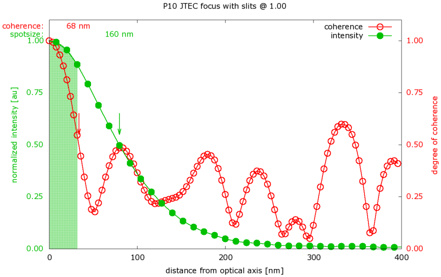
- smaller NA $\longrightarrow$ larger spot
- but only coherent part of the beam

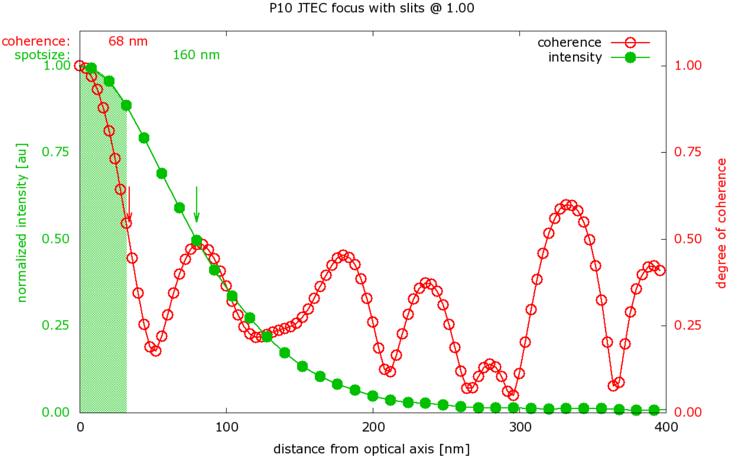
Coherent Flux



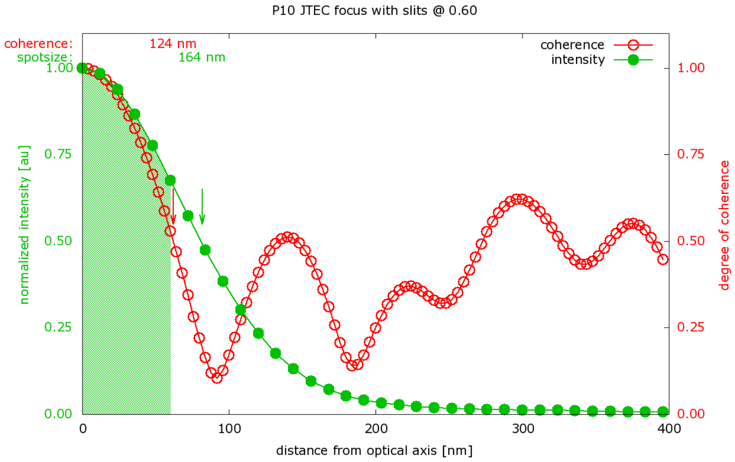
slits in front of the mirror:

- less intensity
- smaller NA $\longrightarrow$ larger spot
- but only coherent part of the beam

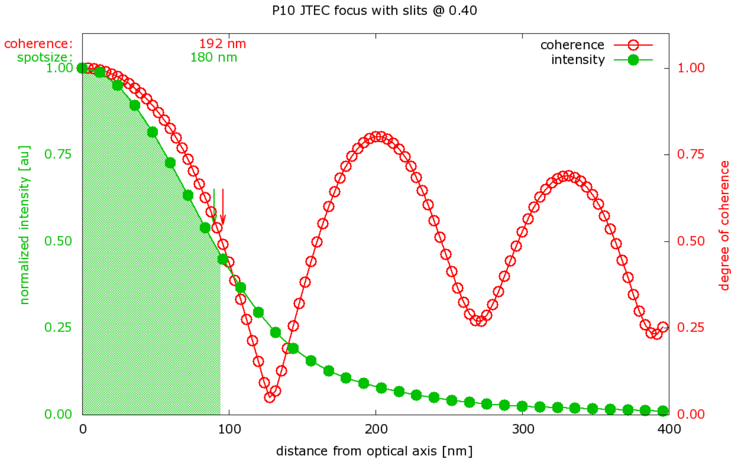
Coherent Flux: Slits @ 1.00



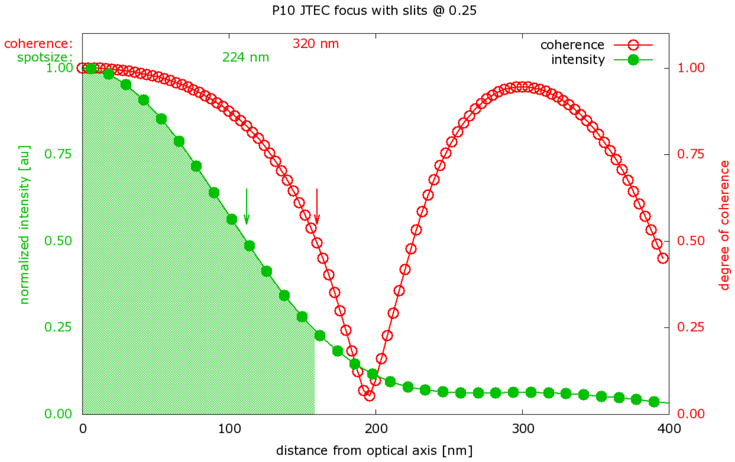
Coherent Flux: Slits @ 0.60



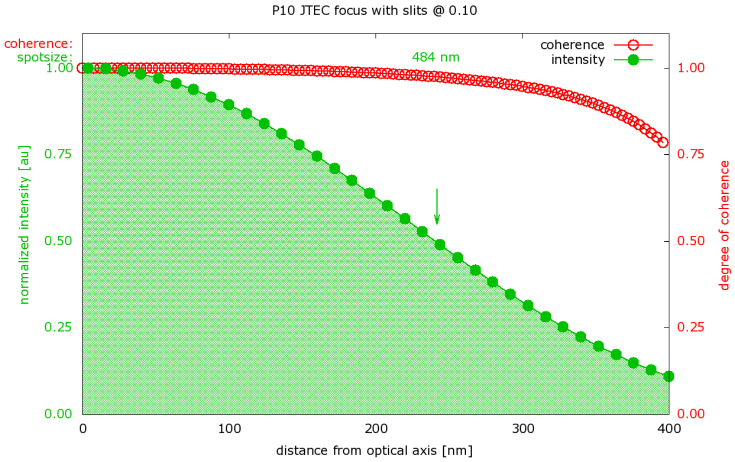
Coherent Flux: Slits @ 0.40



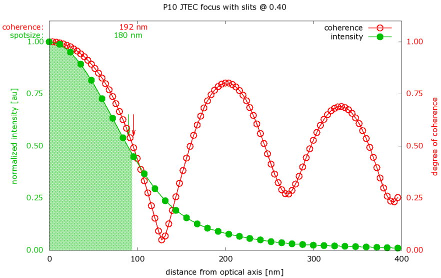
Coherent Flux: Slits @ 0.25



Coherent Flux: Slits @ 0.10

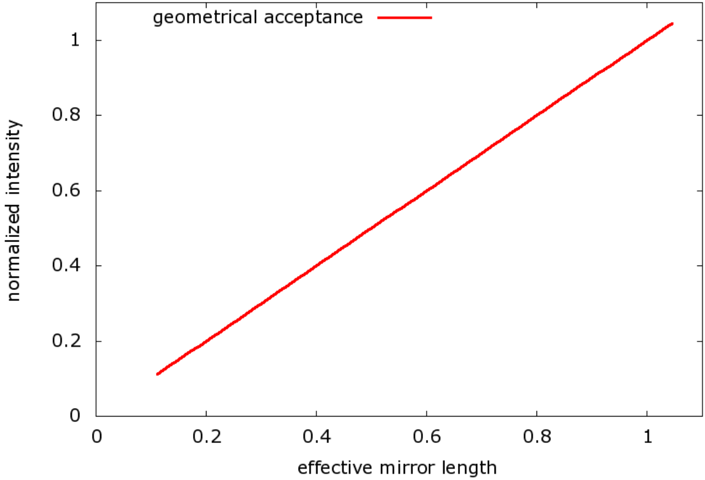


Coherent Flux

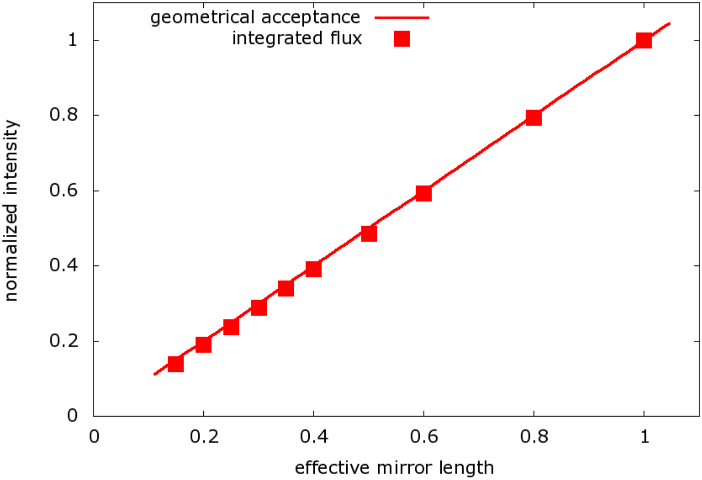


- integrated flux: $\int I(y)\mathrm{d}y$
- coherent flux: integral where $\gamma > 0.5$

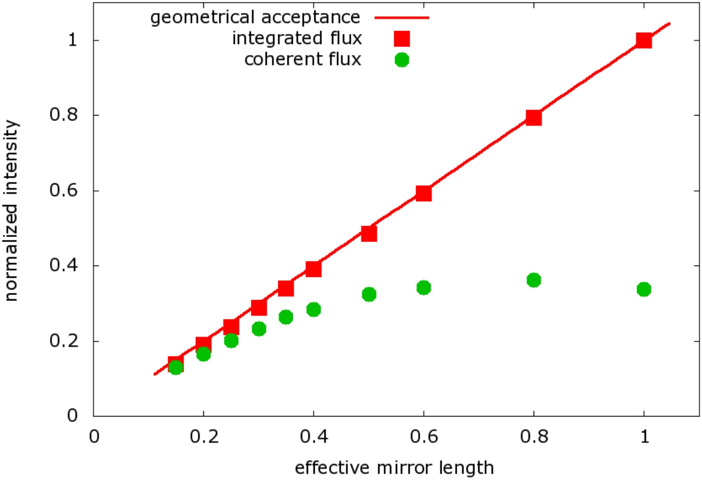
Coherent Flux



Coherent Flux



Coherent Flux



3D-Focus

KB-assumption: two perpendicular foci can be multiplied

- Matlab:

$$\text{imagesc(focus1 } * \text{ transpose(focus2))}$$

- movie: A Flight along the Optical Axis

Summary

- figure error below projected wavelength is not critical
- coherence properties can be enhanced by slits
- optimal ratio coherence / losses wanted

Outlook

- generalization of the source
  - wavelength distribution
  - misalignements, vibrations

  $\longrightarrow$ estimate effect of "beamline errors"
  - emittance

  $\longrightarrow$ calculate gain
  - undulator theory

  $\longrightarrow$ new insight into coherence properties

- generalization from Mirror to Multilayer Mirror
  - development of waveoptical methods in progress
  - estimate effect of inner roughness & errors

  $\longrightarrow$ increase of Numerical Aperture

  $\longrightarrow$ increase of coherence properties?

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Acknowledgements

This transparency is dedicated to...

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  - at the ESRF, Grenoble – Christian Morawe
  - at the University of Göttingen – Tim Salditt
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Thank you for your attention!